Exercise 1

Economic growth: Theory and Empirical Methods, UC3M

Question 1: Consider the Malthus model. Output is produced according to

$$Y(t) = AL(t)^{1-\alpha}, \tag{1}$$

where A is a fixed factor and $L(t)$ is labor in period t. Assume labor dynamics are given by

$$\frac{\dot{L}(t)}{L(t)} = \theta(y(t) - \underline{c}), \tag{2}$$

where y is output per worker and $\underline{c}$ is a subsistence level of consumption.

1. Write output per worker as a function of labor and combine it with the population growth equation to derive a differential equation in labor.

2. Solve for the output per worker in steady state. Explain the intuition behind the result.

3. Use the production function to write the growth rate of output per worker as a function of the growth rate of labor. Substitute in the population dynamics to obtain a differential equation in output per worker.

4. Define $v(t) = \frac{1}{y(t)}$ and rewrite the differential equation into a differential equation in $v(t)$.

5. Solve the differential equation to obtain an explicit solution for $v(t) = \frac{1}{y(t)}$.

6. Use the equation for the growth rate of output per worker to plot it as a function of $y(t)$.

7. Consider an economy with $y(0) > y^*$. Explain the intuition behind the convergence path to the economy's steady state.

Question 2:

1. The R file *students*1 initializes the Malthus model. Plot labor growth on the y-axis against labor on the x-axis. To this end, use $\underline{c} = 500$, $\alpha = 0.5$, $\theta = 0.0002$, and $A = 5000$. To create the x-axis, use the "seq" command from R to create a linear-spaced vector with 100 points between $[0.5L^*, 2L^*]$, where $L^* = (A/\bar{c})^{1/\alpha}$.

2. Plot into the same graph what happens to the labor growth schedule when the fixed factor increases by 30%.

3. Assume $A = 5000$ and, in period 0, we have output per worker $y = 500$ and the amount of labor is $L = 100$. Suppose that because of the black death, labor falls to $L = 50$. Plot output per worker for the next 100 years. To that end, use a "for loop" and compute for each period

- The labor growth rate: $n(t) = \theta(y(t) - \underline{c})$.
- Labor next period: $L(t+1) = (1 + n(t))L(t)$.
- The resulting output per worker.

4. Assume instead that the plague not only decreases labor but also decreases the fixed factor by decreasing productivity: $A = 2500$. Redraw output per worker over time.

5. Use the solution to the differential equation from Question 1 to plot for 100 periods the behavior of $y(t)$ for $y(0) = 800$.